TKE maps and power spectrum agree for $f_e = 400$ Hz. In addition, the organized motions appear fairly consistent throughout the mixing layer. However, if one examines the TKE maps for $f_e = 200$ Hz, the f_p of the organized vortical structures is nominally 300 Hz. This may be a result of the smearing that can occur when phase locking is employed.[5] For the most part f_p estimated from the TKE maps lie within a band of frequencies from 270 to 350 Hz.

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Reynolds Number Effects on the Prediction of Velocity Profile in Compressible Flows

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In a recent article, Huang et al.[1] presented an algorithm for the prediction of skin friction and velocity profiles for compressible turbulent boundary layers. Their method relies on the use of an analytical expression for the mean-velocity profile of compressible turbulent boundary layers, based on the van Driest formulation[2] and the Coles' law of the wake[3]:

$$\frac{u^*}{u_\tau} = \int_0^{y^+} \frac{2}{1+\sqrt{1+4k^2(y^+)^2[1-\exp(-y^+/A^+)]^2}}\,dy^+ + \frac{\Pi}{k} w\left(\frac{y}{\delta}\right) \tag{1}$$

In the above relation, A^+ is the damping factor, k the slope for the law of the wall, $y^+ = yu_\tau/\nu_w$, Π the wake parameter as suggested by Cebecci and Smith[4] $\{\Pi = 0.55[1-\exp(-0.243z_1^{0.5} - 0.298\,z_1)]$, in which $z_1 = Re_\theta/425 - 1\}$, $w(y/\delta) = 1 - \cos[\pi(y/\delta)]$ the wake function, $u_\tau = \sqrt{(\tau_w/\rho_w)}$ the friction velocity, and the transformed velocity is defined by

$$u^* = (u_\delta/b)\sin^{-1}\left\{\left[2b^2(u/u_\delta) - a\right]\Big/(a^2+4b^2)^{0.5}\right\} \tag{2}$$

in which

$$a = (T_\delta/T_w)\left[1 + r(\gamma-1)M_\delta^2/2\right] - 1 \tag{2a}$$

$$b^2 = r(\gamma-1)M_\delta^2(T_\delta/T_w)/2 \tag{2b}$$

where T is the temperature, r the temperature recovery factor, γ the specific heat ratio, M the Mach number, and the subscripts w and δ refer to the conditions at the wall and at the edge of the boundary layer. Then, for a given flow condition with a specific boundary-layer momentum thickness θ (or boundary-layer displacement thickness δ^*) as the only known parameter and by using Eq. (1) and the inverse of the van Driest transformation [Eq. (2)] in an iterative procedure, they demonstrated that a unique solution for the skin friction coefficient and, hence, the velocity profile can be obtained. Although their analysis is based on the validity of the van Driest transformation for general compressible flows and direct extension of the wake-function concept from incompressible turbulent boundary-layer data to compressible flows, it does produce, as was stated by Huang et al.,[5] satisfactory results when applied to a series of experimental data for different freestream Mach numbers, both in terms of skin friction and velocity profile. The speed at which the solution is converged in mathematical sense was also very satisfactory. However, as it will be shown for a given flow Mach number, this method fails to correctly predict the effect of Reynolds number on the mean flow distribution in the boundary layer, and it appears that the accuracy of the method is Reynolds-number dependent. As examples, the effect of Reynolds number on the accuracy of the results can be observed in Figs. 1–6 for three different flow Mach numbers. Experimental data examined here are taken from Winter and Gaudet[6] for $M_\delta = 1.4$ and $Re_\theta = 17\times10^3 \rightarrow 130\times10^3$ (Figs. 1 and 2), $M_\delta = 2.2$ and $Re_\theta = 14\times10^3 \rightarrow 90\times10^3$ (Figs. 3 and 4), and Stalmach[7] for $M_\delta = 3.68$ and $Re_\theta = 2\times10^3 \rightarrow 10.5\times10^3$ (Figs. 5 and 6). As can be seen, there exists strong Reynolds number influence on the prediction of mean-velocity profiles. For all the cases, their method fails to correctly predict the velocity profiles at

Table 1 Percent error (ΔC_f) in the prediction of skin friction coefficient by the method of Huang et al.,[1] $\Delta C_f = (1 - C_{f,\text{calc}}/C_{f,\text{exp}}) \times 100$

	Experimental data			Predicted
Profile	Ref.	M_δ	Re_θ	ΔC_f, %
10	6	1.394	17,914	0.2
11	6	1.395	39,333	1.7
12	6	1.400	60,234	2.0
13	6	1.400	113,948	2.0
14	6	1.400	128,035	2.2
JPL-A112	8	1.314	19,880	2.0
JPL-A113	8	1.321	21,236	2.3
JPL-A114	8	1.320	21,971	2.9
JPL-A115	8	1.315	23,580	2.6
JPL-A122	8	1.308	34,560	2.7
65020301	7	2.865	225,909	2.8
65020401	7	2.897	167,222	2.3
65020501	7	2.908	50,857	2.8
65020601	7	2.910	49,261	3.2
74021801	7	4.517	10,134	−5.4
74021802	7	4.510	15,882	−7.2
74021803	7	4.510	19,789	−5.8
74021804	7	4.500	25,001	−6.1
74021805	7	4.493	28,447	−7.5
58020201	7	2.735	2,011	7.7
58020203	7	2.731	3,834	5.4
58020207	7	2.739	12,228	−5.2
18	6	2.186	14,640	−2.3
19	6	2.197	30,865	−2.3
20	6	2.201	48,223	−2.7
21	6	2.206	88,907	−0.5
39	6	2.198	19,939	−1.9
40	6	2.199	24,993	−2.5
41	6	2.200	30,300	−2.5
42	6	2.201	35,014	−2.2
43	6	2.202	39,867	−2.1
44	6	2.204	46,899	−2.3
45	6	2.205	53,841	−1.5
46	6	2.206	63,569	−1.8
47	6	2.207	73,310	−1.1
48	6	2.208	84,259	−0.4
JPL-A132	8	2.172	23,938	3.5
JPL-A133	8	2.166	24,409	3.3
JPL-A134	8	2.164	25,782	3.2
JPL-A135	8	2.172	25,891	3.0

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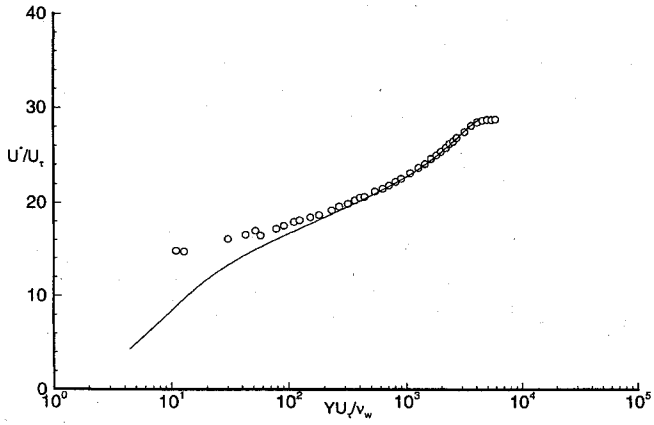

Fig. 1a Transformed velocity profile in semilogarithmic coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 1.3943, Re_θ = 17,914, profile 10: ○, experiment and —, Huang et al.

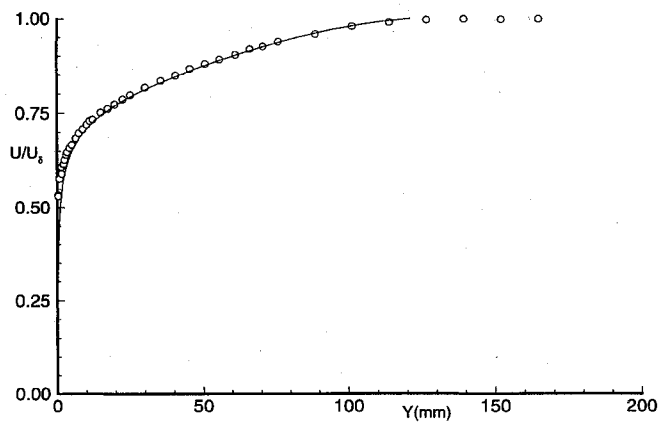

Fig. 1b Untransformed velocity profile in normal coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 1.3943, Re_θ = 17,914, profile 10: ○, experiment and —, Huang et al.

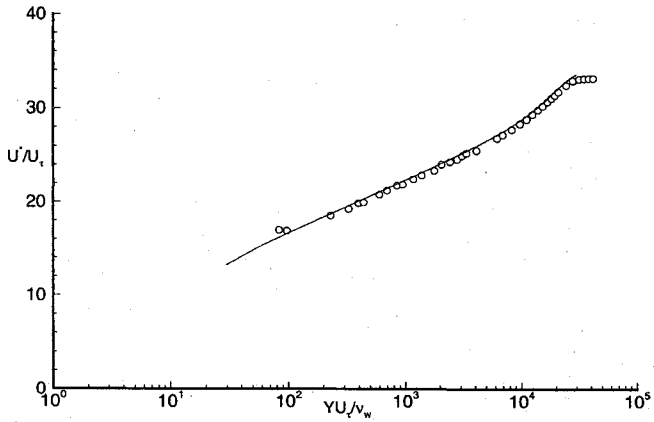

Fig. 2a Transformed velocity profile in semilogarithmic coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 1.4003, Re_θ = 128,035, profile 14: ○, experiment and —, Huang et al.

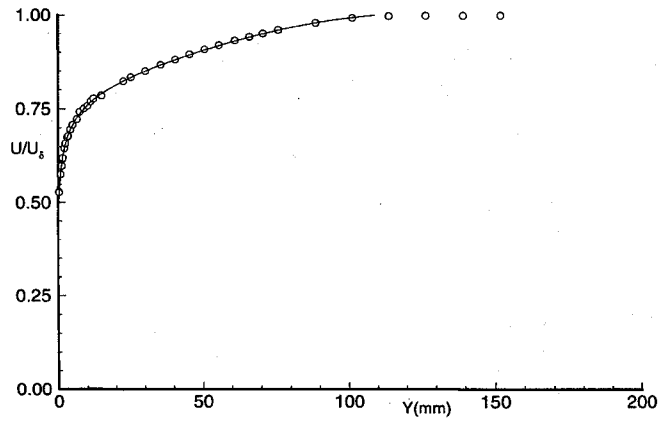

Fig. 2b Untransformed velocity profile in normal coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 1.4003, Re_θ = 128,035, profile 14: ○, experiment and —, Huang et al.

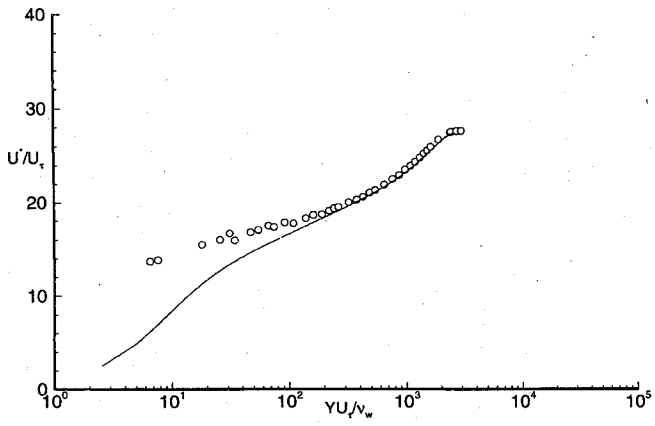

Fig. 3a Transformed velocity profile in semilogarithmic coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 2.1865, Re_θ = 14,640, profile 18: ○, experiment and —, Huang et al.

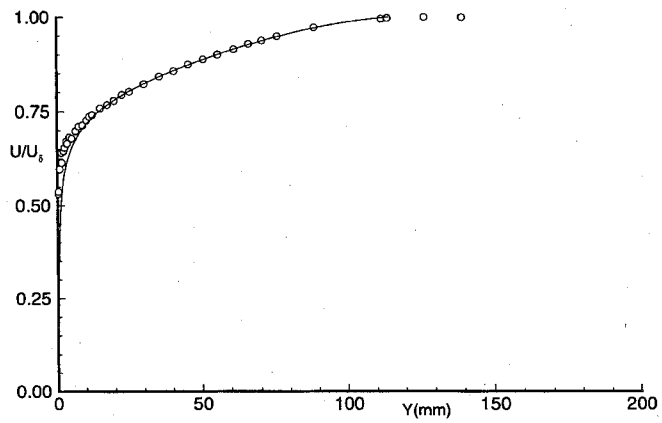

Fig. 3b Untransformed velocity profile in normal coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 2.1865, Re_θ = 14,640, profile 18: ○, experiment and —, Huang et al.

the lower range of Reynolds numbers. But as the Reynolds number increases, the difference between the predicted velocity profile and the experimental data decreases. Note that although the wake parameter Π in the Stalmach's experiment[7] varies from 0.4464 to 0.5499 (low-Reynolds-number effect), this variation does not compensate for the effect of Reynolds number on the whole predicted velocity profile. For the other two sets of data, the wake parameter Π is practically constant and could not influence this behavior (the wake parameter depends strongly on Re_θ only for small Reynolds numbers). The percent error in the prediction of skin friction coefficient ΔC_f [i.e., $\Delta C_f = (1 - C_{f,\text{calc}}/C_{f,\text{exp}}) \times 100$ where $C_{f,\text{calc}}$ and $C_{f,\text{exp}}$ are, respectively, calculated and experimental values of skin friction coefficient] as calculated by their method for a number of experimental data[6–8] is given in Table 1. In contrast to the predicted velocity profile, ΔC_f is not very sensitive to the variation of Re_θ. This strong dependence on the Reynolds number, which is only observed in the predicted velocity profile, suggests that at least for supersonic flows (i.e., $M_\delta > 1.0$) the expression used for

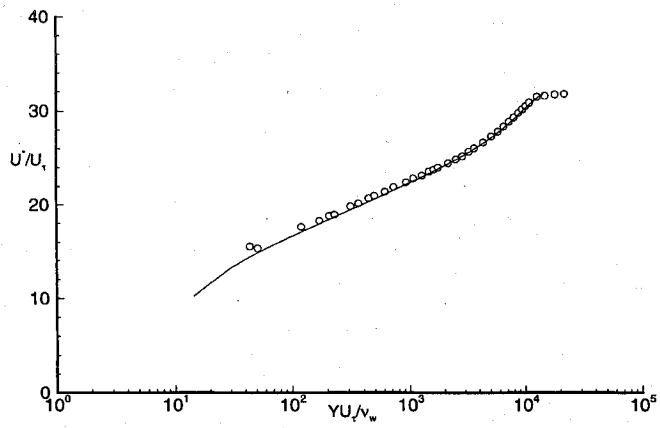

Fig. 4a Transformed velocity profile in semilogarithmic coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 2.2064, Re_θ = 88,907, profile 21: ○, experiment and —, Huang et al.

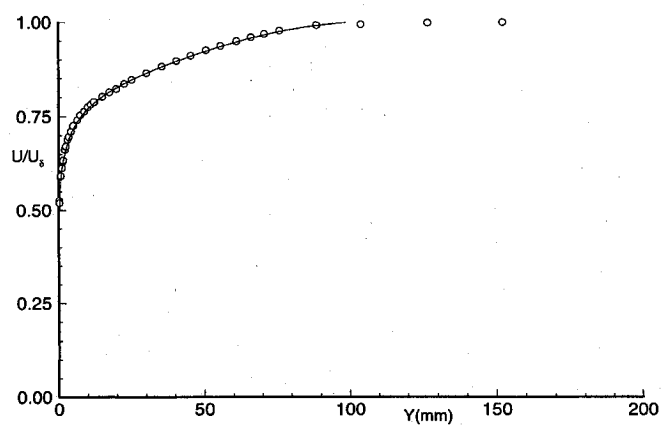

Fig. 4b Untransformed velocity profile in normal coordinates. Experimental data from Winter and Gaudet,[6] M_δ = 2.2064, Re_θ = 88,907, profile 21: ○, experiment and —, Huang et al.

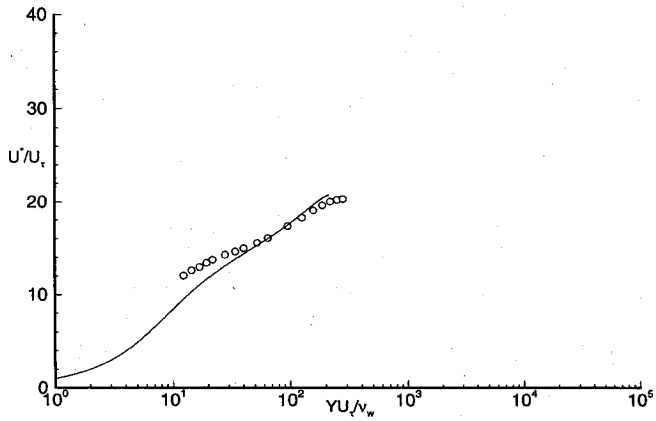

Fig. 5a Transformed velocity profile in semilogarithmic coordinates. Experimental data from Stalmach,[7] M_δ = 3.6840, Re_θ = 2115.3, case 58020301: ○, experiment and —, Huang et al.

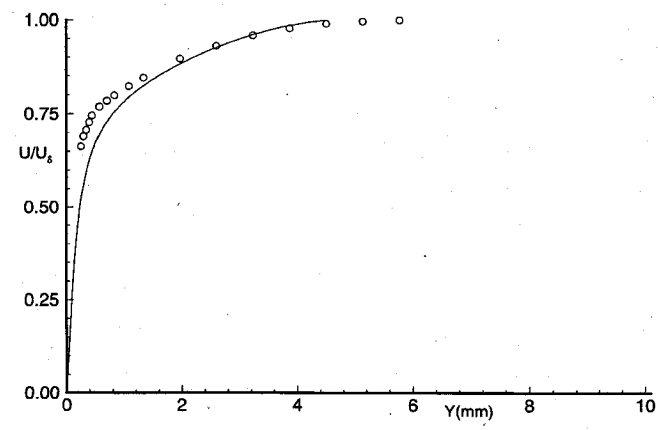

Fig. 5b Untransformed velocity profile in normal coordinates. Experimental data from Stalmach,[7] M_δ = 3.6840, Re_θ = 2115.3, case 58020301: ○, experiment and —, Huang et al.

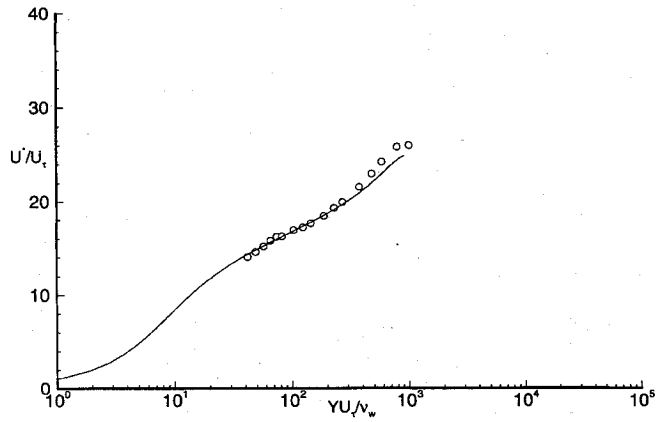

Fig. 6a Transformed velocity profile in semilogarithmic coordinates. Experimental data from Stalmach,[7] M_δ = 3.681, Re_θ = 10,484, case 58020306: ○, experiment and —, Huang et al.

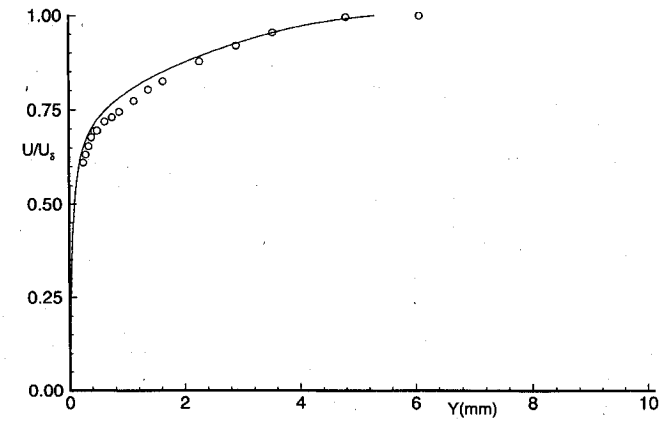

Fig. 6b Untransformed velocity profile in normal coordinates. Experimental data from Stalmach,[7] M_δ = 3.681, Re_θ = 10,484, case 58020306: ○, experiment and —, Huang et al.

the transformed mean-velocity profile of turbulent boundary layers [(i.e., Eq. (1)] does not sufficiently represent all of the physical aspects of the mean flow distribution in a compressible turbulent boundary layer. Extra information,[9,10] or perhaps a more physical approach to the formulation of the mean structure of compressible turbulent boundary layers, is required to achieve complete (physical and mathematical) convergence before it can be applied successfully in any method for the prediction of the mean flow behavior of compressible turbulent boundary layers.

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Reply by the Authors to F. Motallebi

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MOTALLEBI argued that the velocity-profile family of Huang et al.[1] failed to predict the boundary layer at low Reynolds numbers. His argument was based on the observation of experimental data of Winter and Gaudet[6] and Stalmach.[7] In particular, in Figs. 3a ($M_\delta = 2.2$, $Re_\theta = 14{,}640$) and 5a ($M_\delta = 3.7$, $Re_\theta = 2115$), the data close to the wall show higher velocity values than those predicted by the profile family of Huang et al. Motallebi attributed the failure to predict the velocity profile near the wall as "Reynolds number influence" and argued that extra information is needed to explain the low Reynolds number behavior. In his comments, he cited his own reference (Ref. 9) as the source of the explanation for the extra information. But Ref. 9 was simply a letter to the authors to enquire how to get a converged solution with the conditions specified in Huang et al.[1] and the only relevant remark was "More explanations were needed." Now, the velocity profile in a zero-pressure-gradient boundary layer is uniquely defined by any chosen thickness (plus the Reynolds number based thereon) and by the Mach number and heat-transfer parameter. Our velocity-profile family does, of course, need various pieces of empirical information (e.g., the Van Driest transformation and the useful fact that the wake parameter is nearly independent of Mach number). Motallebi apparently failed to observe that although the Reynolds number in Fig. 6a ($M_\delta = 3.7$, $Re_\theta = 10{,}484$) is smaller than that of Fig. 3a, the data in Fig. 6a show a good match to the predicted profile. Note that although the freestream Mach number in Fig. 6a is larger than that in Fig. 3a, the increase of the freestream Mach number tends to reduce the effective Reynolds number (see Huang et al.[1] for further discussion).

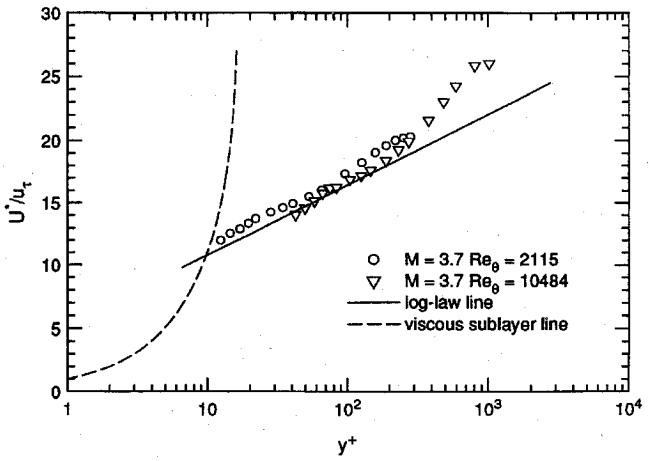

Fig. 1 Transformed velocity profiles (experimental data of Stalmach).

Near the wall (say, $y^+ < 5$), all experimental data have to satisfy

$$\frac{dU^+}{dy^+} = \frac{\mu_w}{\mu} \tag{1}$$

By applying the Van Driest transformation to Eq. (1) and performing the integration from the wall following the assumption that $\mu_w/\mu = (T_w/T)^{0.7}$, one may get a sublayer transformed velocity profile as a function of molecular Prandtl number Pr and two dimensionless parameters

$$M_\tau \left\{ \equiv \frac{u_\tau}{a_w} = \frac{u_\tau}{\sqrt{(\gamma - 1)c_p T_w}} \right\}$$

and

$$B_q \left(\equiv \frac{q_w}{\rho_w c_p u_\tau T_w} \right)$$

(the details of the viscous-sublayer analysis are discussed in Huang and Coleman[2]). Away from the wall ($y^+ > 30$), experimental evidence indicates that the data are good collapse to the incompressible law of the wall line, when the velocity is transformed to the Van Driest variable.

It is difficult to believe that the experimental data quoted by Motallebi, which show log-law regions for y^+ as low as 7 (see Fig. 3a, the lower limit usually quoted is 30–50), are entirely correct. Here, for purpose of illustration, the data of Stalmach are plotted (cases 58020301 and 58020306) in Fig. 1. In the figure, both log-law and viscous-sublayer lines are also shown. In our opinion, the main cause of the discrepancy between data and our predicted profiles is experimental error or scatter—these are rather old data!—rather than an insufficient number of parameters or missing physical information in our analysis, as suggested by Motallebi.

Motallebi seems to overlook many other low Reynolds number compressible flow data included in Ref. 7, which show well-behaved sublayer and buffer-zone characteristics. Perhaps the most interesting comparison of the low Reynolds number compressible near-wall flow can be seen from the paper by Huang and Coleman,[2] in which the data were obtained from a direct numerical simulation of compressible channel flow. Unlike the data shown by Motallebi, the direct numerical simulation data of Huang and Coleman[2] clearly showed three distinct regions—the sub, buffer, and log layers.

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